

2. NUMERICAL METHODS I

**JHU Physics & Astronomy
Python Workshop 2017**

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YOUR NEW FAVOURITE PACKAGE: NUMPY

Numpy provides python with its numerical muscle. **This is your go to package.** The package is written in C and made to deal with N-dimensional arrays, all basic mathematical operations, linear algebra operations, *et cetera*. **We will not be going through all of the power of this module.**

Importing the Numpy module:

```
import numpy as np
```

You will be using numpy for basically all of your future python work. Make sure you import it at the beginning of any/every script and when you first open ipython.

YOUR NEW FAVOURITE PACKAGE: NUMPY

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Importing the Numpy module:

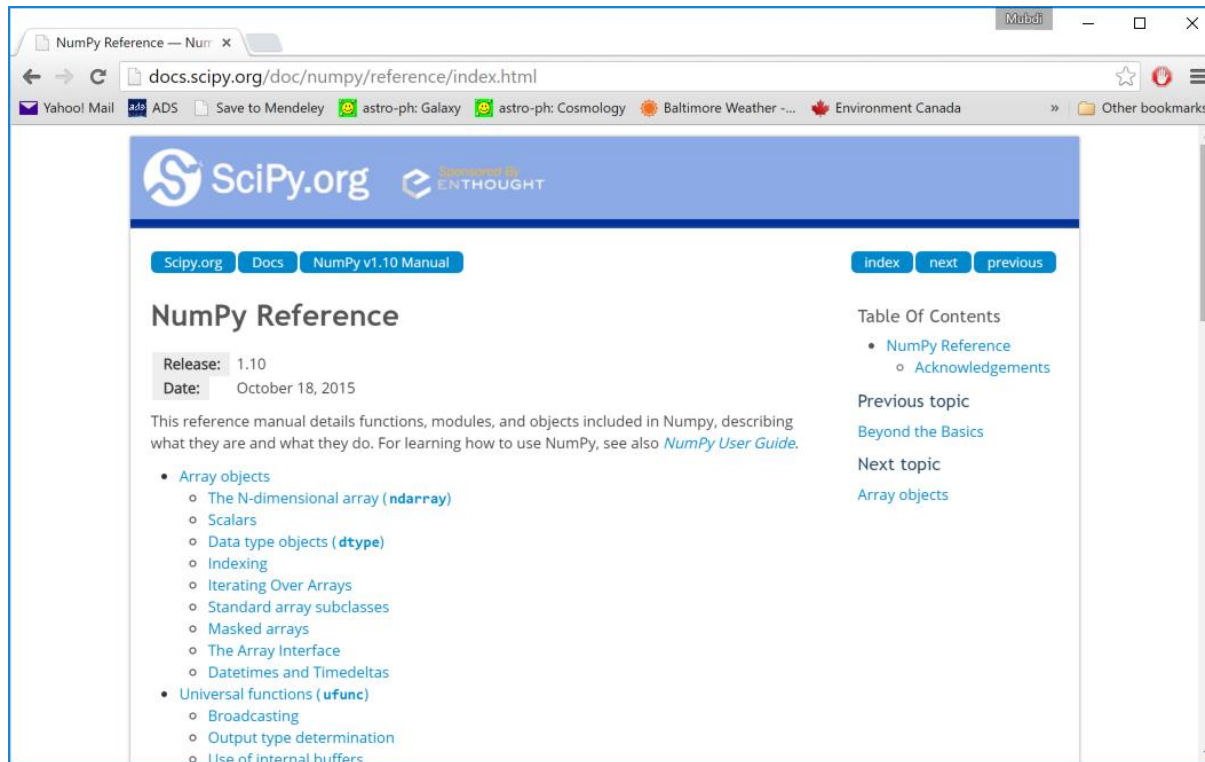
```
import numpy as np
```

You will be using numpy for basic
Make sure you import it at the beginning
when you first open ipython.

MUBDI IS A BONEHEAD NOTE:

I started using python back in the “Wild West” days. Some of the defaults of how I code are not the standards suggested today. In particular, I import numpy as n. Call me on this!

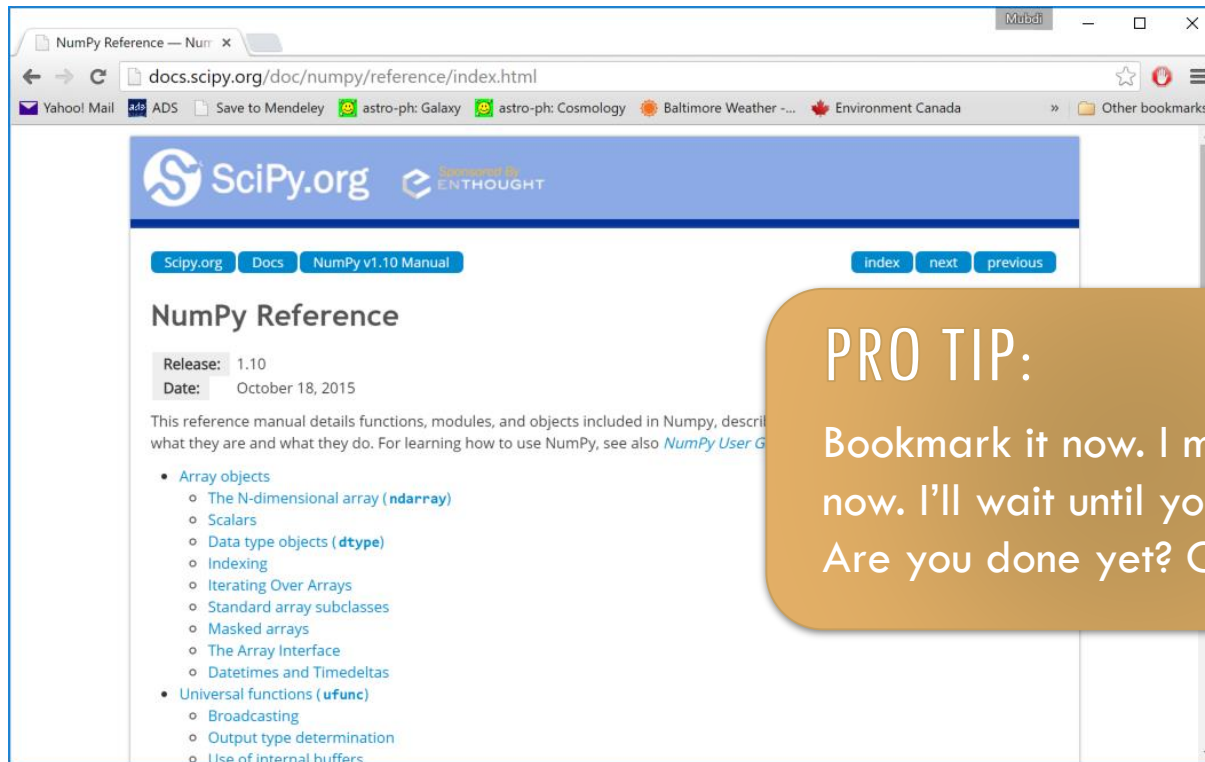
THE DOCUMENTATION



Your reference to all Numpy goodness:

<http://docs.scipy.org/doc/numpy/reference/index.html>

THE DOCUMENTATION



PRO TIP:

Bookmark it now. I mean, right now. I'll wait until you're done... Are you done yet? Good!

Your reference to all Numpy goodness:

<http://docs.scipy.org/doc/numpy/reference/index.html>

NUMPY ARRAYS: YOUR BEST FRIEND

Numpy arrays are the base object containing a variety of powerful methods. Making a Numpy array is easy:

```
In [1]: array1 = np.array([1, 2, 3]) # One-liner
In [2]: list1 = [1.0, 2.0, 3.4]
In [3]: array2 = np.array(list1) # List -> NP Array
```

All data in a Numpy array *must* be of a single data type (dtype). Numpy has a large number of possible data types:

```
np.str # string
np.bool # boolean (i.e., True|False)
np.int # integer
np.float # floating point
np.complex # complex (i.e., 1+1j)
```

NUMPY ARRAYS: YOUR BEST FRIEND

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In [1]: array1 = np.array([1, 2])  
In [2]: list1 = [1.0, 2.0, 3.0]  
In [3]: array2 = np.array(list1)
```

All data in a Numpy array *must* be of the same type.
Numpy has a large number of possible data types:

```
np.str # string  
np.bool # boolean (i.e., True/False)  
np.int # integer  
np.float # floating point  
np.complex # complex (i.e., 1+1j)
```

PRO TIP:

Any of the data type objects have an associated function (with an underscore) to convert a Python array of one sort into another:

```
arr1 = np.array([1, 2])  
arr2 = np.float_(arr1)
```

NUMPY ARRAYS: MULTIDIMENSIONALITY

Numpy arrays can be N-dimensional, which is of particular use with tables of data (i.e. 2-D).

```
# Creating a 3x2 Array:  
array1 = np.array([[1, 2], [3, 4], [5, 6]])  
array1[:,1] # Results in array([2, 4, 6])  
array1.shape # Results in (3, 2)  
array1.size # Results in 6  
  
array1.flatten() # Returns 1-dimensional array  
array1.reshape((1,6)) # Returns a 2-D array
```


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array1.size # Results in 6  
  
array1.flatten() # Returns 1-dim  
array1.reshape((1,6)) # Returns
```

PRO TIP:

There is a difference between a 1-dimensional array and a 2-dimensional array with one of the axes having length 1.

NUMPY ARRAYS: HOW DO YOU MAKE THEM?

Making them manually:

```
# Creating a 2x2 Array:  
array1 = np.array([[1, 2], [3, 4]]) # All ints  
array1 = np.array([[1.0, 2], [3, 4]]) # All floats
```

Or using special functions:

```
# Creating arrays filled sequentially  
array2 = np.arange(10) # [0, 1, 2, ... 9]  
  
# Creating arrays filled with specific values:  
array3 = np.ones(10) # 1-D array filled with 1s  
array4 = np.zeros((3,5)) # 2-D array filled with 0s  
array5 = np.identity(5) # 5x5 identity matrix
```

NUMPY ARRAYS: WHAT CAN YOU DO WITH THEM?

Every Numpy array has lots of built in functionality:

```
# Assuming array1 is a (3,3) numpy array:
```

```
# Basic Parameters:
```

```
array1.min(), array1.max()
```

```
# Taken over one whole array:
```

```
array1.mean(), array1.sum(), array1.prod()
```

```
# Taken in one dimension along first axis:
```

```
array1.mean(axis=0), array1.sum(axis=0)
```

```
# Rearrange data:
```

```
array1.transpose(), array1.swapaxes(0, 1)
```

NUMPY ARRAYS: WHAT CAN YOU DO WITH THEM?

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array1.min(), array1.max()
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```
# Taken over one whole array:
```

```
array1.mean(), array1.sum(), array1.prod()
```

```
# Taken in one dimension along first axis:
```

```
array1.mean(axis=0), array1.sum(axis=0)
```

```
# Rearrange data:
```

```
array1.transpose(), array1.swapaxes(0, 1)
```

PRO TIP:

All this functionality is also provided in functions that take the array as the first argument:

```
np.mean(array1)
```

NUMPY ARRAYS: INDEXING REDUX

```
array1 = np.arange(90).reshape((-1, 10))
```

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 |
| 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 |
| 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 |
| 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 |
| 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 |

NUMPY ARRAYS: INDEXING REDUX

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array1 = np.arange(90).reshape((-1, 10))
```

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 |
| 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 |
| 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 |
| 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 |
| 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 |

PRO TIP:

When reshaping a Numpy array, you can leave one of the axes lengths to be -1, which causes python to determine its length.

NUMPY ARRAYS: INDEXING REDUX

```
array1.shape == (9, 10)
```

10 columns

9 rows



| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 |
| 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 |
| 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 |
| 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 |
| 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 |

NUMPY ARRAYS: INDEXING REDUX

```
array1[5,7] == 57
```

10 columns

9 rows

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 |
| 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 |
| 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 |
| 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 |
| 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 |

NUMPY ARRAYS: INDEXING REDUX

```
array1[5,:] == [50, ... , 59]
```

10 columns

9 rows

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
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NUMPY ARRAYS: INDEXING REDUX

```
array1[5,1:8] == [51, ... , 57]
```

10 columns

9 rows

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
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NUMPY ARRAYS: INDEXING REDUX

```
array1[5:-1,1:8] == [[51, ... , 57], ... , [71, ... , 77]]
```

9 rows

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
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NUMPY ARRAYS: INDEXING REDUX

```
array1[(5, 7), (6, 8)] == [56, 78]
```

Non-sequential indexing!

9 rows

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
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NUMPY ARRAYS: CAUTIONARY TALE

Numpy arrays are generally passed by reference (to minimize space used in memory):

```
In [1]: array1 = np.array([1, 2, 3])
In [2]: array2 = array1[0:2] # Now [1, 2]
In [3]: array1[0] = 5 # Now [5, 2, 3]
In [4]: array2 == np.array([5, 2]) # value edited
```

To ensure that values are independent, use the copy function:

```
In [5]: array2 = np.copy(array1[0:2]) # or
In [6]: array2 = np.array(array1[0:2], copy=True)
```

READING IN TEXT (ASCII) FILES

There are multiple ways of reading in files, but we'll concentrate on the 'loadtxt' function:

```
# Works with a perfectly formatted file  
array1 = np.loadtxt(filename)
```

There are lots of options on this function, so check the docs, but some of the most used:

```
array2 = np.loadtxt(filename, dtype=dtype,  
comments='#', delimiter=',', skiprows=5,  
usecols=(0, 1, 2))  
# This skips all comments (designated with a #) and  
# the first 5 rows. It then reads in columns 0, 1,  
# and 2, delimited by a comma
```

READING IN TEXT (ASCII) FILES

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# This skips all comments (designated by #)  
# the first 5 rows. It then reads the rest of the file  
# and 2, delimited by a comma
```

PRO TIP:

When in doubt about arguments and what form they should be, check the docs:

`np.loadtxt?`

READING IN TEXT (ASCII) FILES

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# the first 5 rows. It then reads in columns 0, 1,  
# and 2, delimited by a comma
```

PRO TIP 2:

Loadtxt can read in gzipped (.gz) and Bzip2 (.bz2) files without them being unzipped.

MATHEMATICAL OPERATIONS

Mathematical operations proceed *element-wise*:

```
# These act on each element  
array1 + 5, array1 * 5, array1**2
```

```
# These act element to element (if the same size)  
array1 + array2, array1*array2
```

And our favourite mathematical operations:

```
# These act on either numpy arrays, floats, or ints  
np.log10(arr1), np.exp(arr1), np.sin(arr1),  
np.cosh(arr1)
```

MATHEMATICAL OPERATIONS

Mathematical operations proceed *element-wise*:

```
# These act on each element  
array1 + 5, array1 * 5, array1**2
```

```
# These act element to element (broadcasting)  
array1 + array2, array1*array2
```

And our favourite mathematical operations:

```
# These act on either numpy array or scalar  
np.log10(arr1), np.exp(arr1), np.sin(arr1),  
np.cosh(arr1)
```

PRO TIP:

Numpy has some useful constants defined, like:

```
np.pi, np.e
```

MATRIX MATH

For 2-D (and higher) matrices, you can do standard matrix math:

```
# Doing standard matrix math:  
np.dot(arr1, arr2), np.cross(arr1, arr2)
```

```
# More complex operations  
np.linalg.eig(arr1), np.linalg.det(arr1)
```

```
# Even calculating inverses:  
np.linalg.inv(arr1)
```

SEARCHING ARRAYS

You can search for specific values in an array using the where command:

```
inds1 = np.where(arr1 > 1)
inds2 = np.where((arr1 > 4) & (arr1 < 10))

# Can use multiple arrays in a single command:
# Arrays must be the same size and shape
inds2 = np.where((arr1 > 4) & (arr2 < 10))
```

The indices given are a tuple with length equalling the number of dimensions of the array. The tuple contains numpy arrays of the index of the matching value in each dimension:

```
inds1 == (array([0, 4, ... , 10])) # One dimension
inds2 == (array([0, ... , 10]), array([3, ... , 5])) #2D
```

SEARCHING ARRAYS

You can search for specific values in an array using the where command:

```
inds1 = np.where(arr1 > 1)
inds2 = np.where((arr1 > 4) & (arr1 < 10))
```

```
# Can use multiple arrays in a single where command
# Arrays must be the same size as the array being searched
inds2 = np.where((arr1 > 4) & (arr2 < 5))
```

The indices given are a tuple with length equal to the number of dimensions of the array. The tuple contains the indices of the matching value in each dimension:

```
inds1 == (array([0, 4, ..., 10])) # One dimension
inds2 == (array([0, ..., 10]), array([3, ..., 5])) #2D
```

PRO TIP:

If there's no values that match your conditions, where will produce a tuple of empty numpy arrays.

VECTORIZING FUNCTIONS

Sometimes, you'll want to make complex functions that don't necessarily automatically work with numpy arrays:

```
def funct1(val):  
    if val > 3: # Doesn't work with array  
        x = 2  
    else:  
        x = 5  
    return x
```

The vectorize function makes functions like this work for arrays:

```
vfunct1 = np.vectorize(funct1)  
vfunct1(arr1) # Now works!
```

VECTORIZING FUNCTIONS

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def funct1(val):  
    if val > 3: # Doesn't work with array  
        x = 2  
    else:  
        x = 5  
    return x
```

The `vectorize` function makes functions like this work:

```
vfunct1 = np.vectorize(funct1)  
vfunct1(arr1) # Now works!
```

PRO TIP:

While this is fine to do for functions you don't need high performance on, it is slow(ish). Consider writing the function better for speed.

SAVING YOUR OUTPUT

For individual numpy arrays, there are some quick and dirty methods to save your data:

```
# Quick and Dirty in Text:  
np.savetxt(filename, arr1)
```

Numpy also has some proprietary formats (.npy, .npz) that allow for quick reading of data:

```
# Saving a single array:  
np.save(filename.npy, arr1)  
  
# Saving multiple arrays:  
np.save(filename.npz, name1=arr1, name2=arr2, ...)
```


SAVING YOUR OUTPUT

For individual numpy arrays, there are some quick and dirty methods to save your data:

```
# Quick and Dirty in Text:  
np.savetxt(filename, arr1)
```

Numpy also has some proprietary formats (like .npy) for quick reading of data:

```
# Saving a single array:  
np.save(filename.npy, arr1)
```

```
# Saving multiple arrays:  
np.save(filename.npz, name1=arr1, name2=arr2, ...)
```

PRO TIP:

Output file extensions are based on how many arrays you have in the save file: .npy is for a single array and .npz is for multiple.

LOADING SAVED OUTPUT

To load a single numpy array (.npy file):

```
arr1 = np.load(filename.npy)
```

To load a multiple numpy arrays (.npz file):

```
alldata = np.load(filename.npz)
```

```
# All Data Object is dictionary-like:
```

```
var1 = alldata['name1']
```

```
var2 = alldata['name2']
```

EXERCISE TIME!

I must have called a
thousand times.